



Public

Report

Document ID 1427105	Version 3.0	Status Approved	Reg no	Page 1 (23)
Author Fredrik Bultmark Katrín Ahlford Mikael Asperö Lind			Date 2014-12-15	
Reviewed by Klas Källström (DR) David Persson (QA)			Reviewed date 2015-08-31 2015-08-31	
Approved by Börje Torstenfelt			Approved date 2015-08-31	
Comment Reviewed according to SKBdoc 1431543				

Radionuclide inventory for application of extension of the SFR repository - Treatment of uncertainties

Innehåll

1	Introduction	3
2	Methodology.....	4
2.1	Radionuclide inventory.....	4
2.1.1	Operational waste.....	4
2.1.2	Correlation factors.....	7
2.1.3	Propagation of uncertainty from measurements to uncertainty factors.....	8
2.1.4	Decommissioning waste.....	9
3	Results.....	10
3.1	Silo Vault.....	10
3.2	BRT Vault.....	11
3.3	1BMA Vault.....	12
3.4	2BMA Vault.....	13
3.5	1BTF Vault.....	14
3.6	2BTF Vault.....	15
3.7	1BLA Vault.....	16
3.8	XBLA Vaults.....	17
3.9	All vaults.....	18
	References	19
	Appendix A	20
A.1	Normally distributed random variables	20
A.2	Log-normally distributed variables.....	20
A.3	The sum of two continuous stochastic variables	21
A.3.1	Sum of two normally distributed variables	21
A.4	The product of two continuous stochastic variables.....	21
A.4.1	The product of two log-normally distributed variables.....	21
A.5	Mapping a normal distribution onto a log-normal distribution	22

1 Introduction

The reference radionuclide inventory is presented in the report SKB 2013, where several different methods for estimating the activity of the most important radionuclides in the SFR inventory have been applied depending on the type and origin of the waste. Each method is associated with an uncertainty. This report therefore presents a reference inventory and an estimated maximum inventory which would be valid if the considered uncertainties are evaluated at a confidence level of 95%. The waste packages are assumed to be placed in the repository vaults according to the deposition strategy presented in Gordon (2014).

2 Methodology

2.1 Radionuclide inventory

2.1.1 Operational waste

The activity in the operational waste is estimated by measurements and calculations as well as correlation to key radionuclides. The key radionuclides Co-60 and Cs-137 can be measured directly on each waste package using gamma detectors. The uncertainty in the gamma measurements can vary depending on whether the waste is homogeneous or heterogeneous (Bäcklin 1990).

Certain actinides as well as Sr-90 are measured using alpha or scintillation detectors on water samples from different process systems at the nuclear power plants and Clab. The uncertainties are also in this case dependent on whether the waste type is homogeneous or not (Ingemansson 2000a). For the nuclear power plants and Clab, the measured sum of Pu-239/240 is used as a key nuclide for correlation of other uranic or transuranic isotopes.

Since a few years, the nuclear power plants (except from BKAB) have started measuring Ni-63, which is analyzed using scintillation detectors. The Ni-59 amount is thereafter determined by correlation to Ni-63. Even if this procedure results in smaller uncertainties, the number of years where correlation of nickel to Co-60 has been used outweighs the few years nickel measurements have been performed. Therefore, the uncertainty in correlation is used in this report for the nickel isotopes.

For the nuclear power plants, the radionuclides C-14, Cl-36 (only in waste containing ion exchange resin), Mo-93, Tc-99, I-129 and Cs-135 are calculated from certain calculation models based on reactor water measurements and other parameters such as the nuclear power plant energy production. The model for Cl-36 is considered to be conservative and no additional uncertainty is needed. For Clab, an adjusted calculation is performed to determine the amount of C-14, Mo-93, Tc-99, I-129 and Cs-135, whereas Cl-36 is determined by correlation to Co-60. For SNAB and Svafo, there is also an adjusted calculation available for Mo-93, Tc-99, I-129 and Cs-135, whereas all the remaining radionuclides are determined by correlation to the measured key nuclides Co-60 and Cs-137.

Radionuclides that are neither measured directly on the produced waste packages, nor correlated to key radionuclides are being reported as a total produced activity each year. Thereafter, the activity is distributed over waste packages that are disposed of in SFR according to different distribution models. There is an uncertainty coupled to the distribution procedure, which has been applied for all non-waste package specific data in this report.

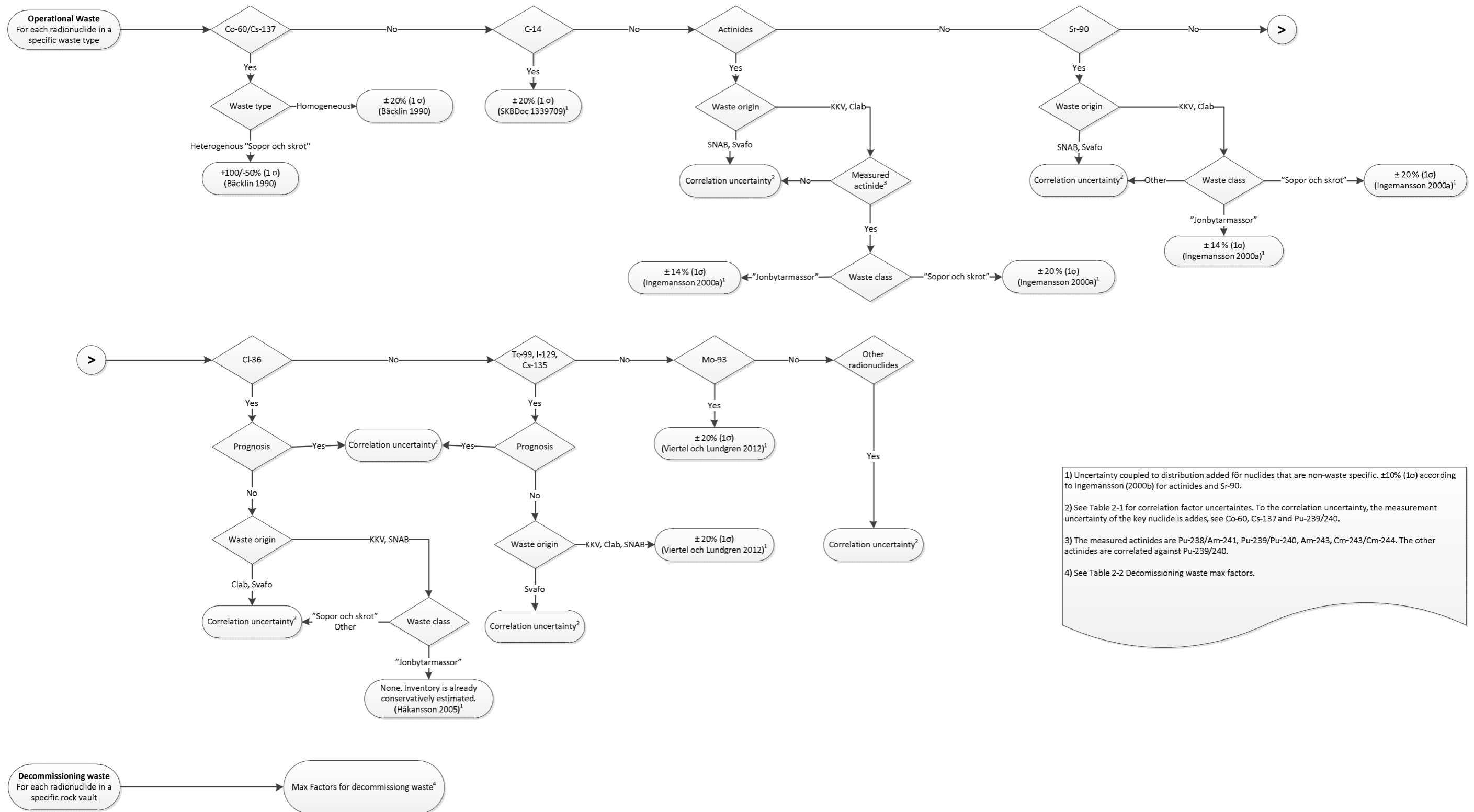
For the radionuclides Cl-36, Tc-99, I-129 and Cs-135, different methods are applied depending on whether the waste has already been disposed of, or if it is part of the prognosis. For these radionuclides, the uncertainty therefore varies.

Table 2-1 summarizes the radionuclide specific uncertainties that are considered in the following calculations, whereas Figure 2-1 also illustrates how the methods for estimation differ depending on the type and origin of the waste. In addition to the uncertainties associated with the methods for estimation, there are also uncertainties coupled to the future waste production, e.g. the amount of waste packages that will be disposed of in SFR before closure. These uncertainties have, however, not been considered in the calculations in this report.

Table 2-1 Uncertainties (1σ) in inventory of operational waste. Values from (SKB 2013)

Nuclide/method	Uncertainty [%]	Uncertainty
Co-60 and Cs-137 in homogeneous waste	20	Uncertainty in gamma measurement on waste package
Co-60 and Cs-137 in heterogeneous waste	+100/-50	Uncertainty in gamma measurement on waste package
Actinides and Sr-90 in homogeneous waste	14	Uncertainty in nuclide specific model of yearly production based on measurements of water samples
Actinides and Sr-90 in heterogeneous waste	20	Uncertainty in nuclide specific model of yearly production based on measurements of water samples
C-14	20	Uncertainty in model of yearly production based on measurements on ion exchange resin samples
Cl-36 in ion exchange resin	¹	Uncertainty in model of yearly production based on activation calculation
Mo-93, Tc-99, I-129 and Cs-135	20	Uncertainty in model of yearly production based on activation calculation
Distribution models	10	Uncertainty in distribution of activity following mass of ion exchange resin or Co-60/Cs-137 activity

¹ The model used to calculate the activity of Cl-36 is estimated to be conservative, and thus no additional uncertainty is needed here.



1) Uncertainty coupled to distribution added for nuclides that are non-waste specific. ±10% (1σ) according to Ingemansson (2000b) for actinides and Sr-90.
 2) See Table 2-1 for correlation factor uncertainties. To the correlation uncertainty, the measurement uncertainty of the key nuclide is added, see Co-60, Cs-137 and Pu-239/240.
 3) The measured actinides are Pu-238/Am-241, Pu-239/Pu-240, Am-243, Cm-243/Cm-244. The other actinides are correlated against Pu-239/240.
 4) See Table 2-2 Decommissioning waste max factors.

Figure 2-2 Radionuclide uncertainty flow sheet. KKV means the nuclear power plants

2.1.2 Correlation factors

Correlation factors are generally used when there is no other method for estimating the radionuclide, but are in some cases also used when the waste has been deemed having a low contribution to the total activity (for instance waste from Studsvik and Clab), see Figure 2-1. The method uses the measured activity of either Co-60, Cs-137 or Pu-239/240 on each waste package and then correlates that measurement to the wanted radionuclide using factors produced in Lindgren et al. (2007) with exception for Tc-99 (Lundgren 2010) and Pu-241 (Thierfeldt et al. 1995).

The factors are produced using measurements of the wanted radionuclide correlated with measurements of Co-60, Cs-137 or Pu-239/240. Measurements from both Swedish and other nuclear power plants are used and thus, the correlation factors come with large uncertainties. The available data has large uncertainties as they are based on measurements of different types of materials (fuel, core, coolant water, resin etc.) and on waste from different reactor types (BWR and PWR). There are also other differences in reactor operations and chemistry of coolant which adds to the general uncertainty of the correlation factors (Lindgren et al. 2007).

In Forsyth (1997) and Cronstrand (2005 and 2007) analyses were performed on the background data of the correlation factors in order to be able to quantify the uncertainties into statistical distributions. The two Cronstrand reports use similar methods: For each radionuclide the available data is evaluated using standard statistical methods and if possible a logarithmic mean and standard deviation is calculated. After that a qualitative assessment is performed to evaluate if the data is representative for SKB use and the amount of data is sufficient. The result of this qualitative assessment is a semi quantitative uncertainty factor of at least 95% confidence. The 2007 report improves on some of the radionuclides uncertainty factors given in the 2005 report by using new data taken strictly from the Swedish nuclear power plants.

In Table 2-2 the correlation factor uncertainties used in this report are presented.

Table 2-2 Correlation factor (at 1.645 σ) uncertainties with references

Radionuclide	Correlates with ¹	Uncertainty factor at 1.645 σ	Reference
H-3	Co-60	50,00	Cronstrand 2005
Be-10	Co-60	50,00	Cronstrand 2005
Cl-36	Co-60	6,00	Cronstrand 2005
Fe-55	Co-60	5,00	Cronstrand 2005
Ni-59	Co-60	3,00	Cronstrand 2005
Ni-63	Co-60	3,00	Cronstrand 2005
Se-79	Cs-137	50,00	Cronstrand 2005
Sr-90	Cs-137	5,00	Cronstrand 2005
Zr-93	Co-60	50,00	Cronstrand 2005
Nb-93m	Co-60	20,00	Cronstrand 2005
Nb-94	Co-60	5,00	Cronstrand 2005
Tc-99	Co-60	5,00	Cronstrand 2005
Pd-107	Cs-137	40,00	Cronstrand 2005
Ag-108m	Co-60	50,00	Cronstrand 2005
Cd-113m	Cs-137	50,00	Cronstrand 2005
Sn-126	Cs-137	40,00	Cronstrand 2005
Sb-125	Co-60	10,00	Cronstrand 2005
I-129	Cs-137	5,00	Cronstrand 2005
Ba-133	Co-60	2,00	SKB 2013
Cs-134	Cs-137	1,20	SKB 2013
Cs-135	Cs-137	3,00	Cronstrand 2005
Pm-147	Cs-137	2,00	SKB 2013
Sm-151	Cs-137	2,00	SKB 2013
Eu-152	Cs-137	2,00	SKB 2013

Eu-154	Cs-137	2,00	SKB 2013
Eu-155	Cs-137	2,00	SKB 2013
Ho-166m	Co-60	2,00	SKB 2013
U-232	Pu-239/240	2,00	SKB 2013
U-234	Pu-239/240	2,00	SKB 2013
U-235	Pu-239/240	2,00	SKB 2013
U-236	Pu-239/240	2,00	SKB 2013
U-238	Pu-239/240	2,00	SKB 2013
Np-237	Pu-239/240	2,00	SKB 2013
Pu-238	Pu-239/240	2,00	SKB 2013
Pu-241	Pu-239/240	2,00	SKB 2013
Pu-242	Pu-239/240	2,00	SKB 2013
Am-241	Pu-239/240	10,00	Cronstrand 2005
Am-242m	Pu-239/240	2,00	SKB 2013
Am-243	Pu-239/240	2,00	SKB 2013
Cm-243	Pu-239/240	2,00	SKB 2013
Cm-244	Pu-239/240	2,00	SKB 2013
Cm-245	Pu-239/240	2,00	SKB 2013
Cm-246	Pu-239/240	2,00	SKB 2013

¹ Waste from SNAB and Svafo correlates actinides against Cs-137.

2.1.3 Propagation of uncertainty from measurements to uncertainty factors

The measurement uncertainty of the nuclides Co-60, Cs-137 and Pu-239/240 are viewed as normally distributed when summed with other normally distributed uncertainties. The combined uncertainty factor is then calculated as

$$k_z \frac{\Delta z}{z} = \sqrt{\left(k_x \frac{\Delta x}{x}\right)^2 / \max(1, N) + \left(k_y \frac{\Delta y}{y}\right)^2} \quad \text{Eq 1}$$

Where $k_i = 1.645$ for a 95 % confidence level and N is the number of deposited packages of this particular waste type. The absolute error is then calculated using the reported mean value for the radionuclide times the total uncertainty calculated above. The sum of the mean value and the absolute error is used for the maximum inventory.

In case the measurement uncertainty is to be combined with a correlation uncertainty, the normal distribution of the measurement uncertainty is mapped on a log-normal distribution with the same $x_{50\%}$ (median) and $x_{95\%}$. A detailed description of this process is found in Appendix A. The combined uncertainty is

$$z + \Delta z|_k = z \cdot e^{\sqrt{\left[\ln\left(1 + k_x \frac{\Delta x}{x} \Big|_{1\sigma} / \sqrt{N}\right)\right]^2 + \left[\ln\left(1 + k_y \frac{\Delta y}{y} \Big|_{1\sigma}\right)\right]^2}} \quad \text{Eq 2}$$

2.1.4 Decommissioning waste

The radionuclide inventory for decommissioning waste has been determined using calculation models. Depending on where the material in the waste packages comes from, the nuclide specific activity is calculated and given an uncertainty. These amounts are reported together with maximum and minimum uncertainty factors for each radionuclide (SKB 2013, Appendix D). The maximum uncertainty factors are adjusted to a confidence level of 95% and are used to calculate the maximum radionuclide inventory from decommissioning waste. The maximum uncertainty factors are rock vault specific and are presented in Table 2-3. The radionuclide inventory in the decommissioning waste from SNAB and Svafo has not been estimated to date, which is why these amounts are assumed to be included in their prognosis for operational waste.

Table 2-3 Max uncertainty factors (2 σ) for decommissioning waste (SKB 2013 Appendix D). Empty cells indicate no activity from that nuclide in the decommissioning waste in that particular rock vault.

Radionuclide	Max factor silo (2 σ)	Max factor BRT (2 σ)	Max factor 1-2BMA (2 σ)	Max factor 1-5BLA (2 σ)
H-3			7,19	3,17
Be-10			4,71	3,00
C-14	9,73	2,00	3,99	6,31
Cl-36	9,39	2,00	4,27	2,72
Ca-41			4,52	2,91
Fe-55	5,01	2,00	2,53	6,99
Co-60	5,16	2,00	2,20	7,81
Ni-59	4,18	2,00	2,32	7,37
Ni-63	4,20	2,00	2,31	7,42
Se-79	9,63		9,25	9,69
Sr-90	4,42	2,00	2,52	9,05
Zr-93	4,87	2,00	2,45	7,04
Nb-93m	4,65	2,00	2,10	7,65
Nb-94	4,59	2,00	2,17	7,38
Mo-93	4,68	2,00	2,17	6,23
Tc-99	8,07	2,00	3,21	9,17
Pd-107	9,79		2,00	8,16
Ag-108m	4,73	2,00	2,39	4,60
Cd-113m	9,78		3,30	9,06
In-115			3,00	
Sn-126	9,50	2,00	2,48	8,19
Sb-125	5,02	2,00	2,42	7,00
I-129	9,68		10,00	9,05
Cs-134	9,88		9,29	8,00
Cs-135	9,69		11,00	8,20
Cs-137	9,71		10,68	8,44
Ba-133	9,82		4,51	3,00
Pm-147	5,61	2,00	2,03	8,42
Sm-151	4,42	2,00	4,63	3,04
Eu-152	4,76	2,00	4,14	3,00
Eu-154	4,71	2,00	3,41	3,41
Eu-155	5,03	2,00	3,12	3,34
Ho-166m	4,85	2,00	5,27	3,00
U-232	5,19	2,00	2,01	8,47
U-235	4,33	2,00	2,35	7,68
U-236	4,98	2,00	1,98	8,36
Np-237	4,38	2,00	1,98	7,94
Pu-238	4,38	2,00	1,98	7,93
Pu-239	4,31	2,00	2,20	7,80
Pu-240	4,31	2,00	2,10	7,99
Pu-241	4,36	2,00	1,92	8,10
Pu-242	4,33	2,00	1,97	7,99
Am-241	4,37	2,00	2,03	7,96
Am-242m	4,21	2,00	1,96	7,84
Am-243	4,38	2,00	1,98	8,01
Cm-243	4,35	2,00	1,97	7,96
Cm-244	4,46	2,00	1,98	8,08
Cm-245	4,47	2,00	2,04	7,87
Cm-246	4,49	2,00	2,03	8,03

3 Results

3.1 Silo Vault

The proposed Silo inventory is presented in Table 3-1. A comparison with the reference inventory (SKB 2013) is also given in the same table. For individual waste packages, see Appendix B.

Table 3-1 Silo Inventory

Radionuclide	Reference inventory [Bq]	Maximum inventory [Bq]	Uncertainty factor (1.645 σ)
H-3	8,97E+09	4,56E+11	50,88
Be-10	9,89E+05	4,98E+07	50,33
C-14 org	7,55E+11	8,87E+11	1,17
C-14 oorg	2,72E+12	3,19E+12	1,17
Cl-36	8,94E+08	3,91E+09	4,38
Fe-55	2,73E+12	1,52E+13	5,55
Co-60	1,29E+13	2,40E+13	1,86
Ni-59	6,86E+12	2,09E+13	3,05
Ni-63	5,47E+14	1,67E+15	3,05
Se-79	1,05E+09	5,31E+10	50,45
Sr-90	3,61E+12	4,76E+12	1,32
Zr-93	4,48E+09	1,00E+11	22,38
Nb-93m	9,31E+12	3,99E+13	4,28
Nb-94	8,65E+10	3,60E+11	4,17
Mo-93	1,96E+10	4,22E+10	2,16
Tc-99	5,00E+10	1,94E+11	3,88
Pd-107	2,75E+08	1,07E+10	39,00
Ag-108m	2,30E+11	5,08E+12	22,12
Cd-113m	9,58E+09	5,03E+11	52,50
Sn-126	2,05E+08	5,90E+09	28,79
Sb-125	1,32E+11	1,54E+12	11,65
I-129	9,84E+08	3,18E+09	3,24
Cs-134	2,20E+11	5,85E+11	2,66
Cs-135	4,47E+09	9,40E+09	2,10
Cs-137	5,97E+13	9,00E+13	1,51
Ba-133	6,16E+08	1,36E+09	2,21
Pm-147	3,58E+11	1,18E+12	3,28
Sm-151	4,63E+11	1,03E+12	2,23
Eu-152	8,64E+08	2,29E+09	2,65
Eu-154	5,24E+11	1,48E+12	2,82
Eu-155	9,96E+10	3,17E+11	3,18
Ho-166m	6,82E+09	1,41E+10	2,06
U-232	6,20E+05	1,54E+06	2,48
U-234	3,58E+07	8,24E+07	2,31
U-235	1,42E+07	2,86E+07	2,02
U-236	1,58E+07	4,04E+07	2,55
U-238	3,28E+07	6,99E+07	2,13
Np-237	5,36E+08	1,60E+09	2,98
Pu-238	7,29E+10	1,75E+11	2,39
Pu-239	1,70E+10	3,42E+10	2,01
Pu-240	2,39E+10	4,83E+10	2,02
Pu-241	3,07E+11	7,71E+11	2,51
Pu-242	1,23E+08	3,06E+08	2,49
Am-241	2,32E+13	2,81E+14	12,13
Am-242m	3,22E+08	8,22E+08	2,56
Am-243	1,59E+09	3,09E+09	1,94
Cm-243	1,89E+08	4,07E+08	2,16
Cm-244	9,26E+09	2,61E+10	2,81
Cm-245	1,49E+07	4,08E+07	2,74
Cm-246	4,29E+06	1,22E+07	2,84
Total	6,71E+14	2,16E+15	3,22

3.2 BRT Vault

The proposed BRT inventory is presented in Table 3-2. A comparison with the reference inventory (SKB 2013) is also given in the same table. For individual reactor tanks, see Appendix B.

Table 3-2 BRT Inventory

Radionuclide	Reference inventory [Bq]	Maximum inventory [Bq]	Uncertainty factor (1.645 σ)
C-14 ind	1,02E+10	1,85E+10	1,82
Cl-36	7,20E+06	1,31E+07	1,82
Fe-55	1,49E+10	2,72E+10	1,82
Co-60	1,93E+11	3,51E+11	1,82
Ni-59	1,60E+11	2,91E+11	1,82
Ni-63	1,44E+13	2,62E+13	1,82
Sr-90	2,32E+10	4,23E+10	1,82
Zr-93	1,84E+08	3,36E+08	1,82
Nb-93m	1,06E+12	1,92E+12	1,82
Nb-94	7,94E+09	1,45E+10	1,82
Mo-93	2,99E+09	5,46E+09	1,82
Tc-99	4,50E+08	8,19E+08	1,82
Ag-108m	1,62E+09	2,96E+09	1,82
Sn-126	7,52E+05	1,37E+06	1,82
Sb-125	1,34E+07	2,45E+07	1,82
Pm-147	1,37E+06	2,49E+06	1,82
Sm-151	3,42E+08	6,23E+08	1,82
Eu-152	5,41E+05	9,86E+05	1,82
Eu-154	9,26E+07	1,69E+08	1,82
Eu-155	2,40E+06	4,37E+06	1,82
Ho-166m	8,00E+03	1,46E+04	1,82
U-232	6,86E+03	1,25E+04	1,82
U-235	1,49E+01	2,71E+01	1,82
U-236	3,92E+05	7,14E+05	1,82
Np-237	4,70E+05	8,57E+05	1,82
Pu-238	2,71E+09	4,95E+09	1,82
Pu-239	4,16E+08	7,57E+08	1,82
Pu-240	5,93E+08	1,08E+09	1,82
Pu-241	9,06E+09	1,65E+10	1,82
Pu-242	3,11E+06	5,67E+06	1,82
Am-241	1,99E+09	3,62E+09	1,82
Am-242m	1,32E+07	2,40E+07	1,82
Am-243	4,14E+07	7,54E+07	1,82
Cm-243	6,38E+06	1,16E+07	1,82
Cm-244	6,74E+08	1,23E+09	1,82
Cm-245	6,82E+05	1,24E+06	1,82
Cm-246	2,24E+05	4,08E+05	1,82
Total	1,59E+13	2,89E+13	1,82

3.3 1BMA Vault

The proposed 1BMA inventory is presented in Table 3-3. A comparison with the reference inventory (SKB 2013) is also given in the same table. For individual waste packages, see Appendix B.

Table 3-3 1BMA Inventory

Radionuclide	Reference inventory [Bq]	Maximum inventory [Bq]	Uncertainty factor (1.645 σ)
H-3	8,09E+08	4,06E+10	50,18
Be-10	2,21E+05	1,11E+07	50,06
C-14 org	1,47E+11	1,71E+11	1,17
C-14 oorg	1,89E+12	2,21E+12	1,17
Cl-36	3,34E+08	6,67E+08	2,00
Fe-55	5,35E+10	2,72E+11	5,08
Co-60	4,08E+11	4,74E+11	1,16
Ni-59	2,10E+12	6,30E+12	3,00
Ni-63	1,47E+14	4,40E+14	3,00
Se-79	2,10E+08	1,05E+10	50,02
Sr-90	5,49E+11	6,61E+11	1,20
Zr-93	3,68E+08	1,84E+10	50,06
Nb-93m	1,73E+10	3,47E+11	20,08
Nb-94	3,67E+09	1,84E+10	5,01
Mo-93	1,46E+09	2,00E+09	1,37
Tc-99	6,22E+09	1,11E+10	1,79
Pd-107	5,24E+07	2,10E+09	40,02
Ag-108m	1,95E+10	9,75E+11	50,06
Cd-113m	7,98E+08	3,99E+10	50,07
Sn-126	2,62E+07	1,05E+09	40,02
Sb-125	4,37E+07	4,61E+08	10,55
I-129	1,46E+08	2,81E+08	1,92
Cs-134	1,45E+08	1,93E+08	1,33
Cs-135	8,42E+08	1,13E+09	1,34
Cs-137	8,15E+12	8,35E+12	1,02
Ba-133	4,89E+07	9,99E+07	2,04
Pm-147	3,71E+08	7,63E+08	2,06
Sm-151	8,27E+10	1,66E+11	2,00
Eu-152	9,47E+07	2,07E+08	2,19
Eu-154	2,33E+10	4,71E+10	2,02
Eu-155	1,02E+09	2,07E+09	2,03
Ho-166m	1,41E+09	2,83E+09	2,01
U-232	8,85E+04	1,92E+05	2,17
U-234	6,66E+06	1,41E+07	2,12
U-235	3,00E+06	6,01E+06	2,01
U-236	2,65E+06	5,53E+06	2,09
U-238	5,95E+06	1,22E+07	2,05
Np-237	2,72E+07	5,48E+07	2,01
Pu-238	7,51E+09	1,23E+10	1,64
Pu-239	2,78E+09	3,80E+09	1,37
Pu-240	3,87E+09	5,26E+09	1,36
Pu-241	2,40E+10	5,80E+10	2,42
Pu-242	2,00E+07	4,24E+07	2,12
Am-241	2,91E+10	7,16E+10	2,46
Am-242m	4,46E+07	9,54E+07	2,14
Am-243	2,02E+08	2,74E+08	1,36
Cm-243	1,85E+07	2,98E+07	1,62
Cm-244	6,73E+08	1,42E+09	2,12
Cm-245	1,99E+06	4,21E+06	2,12
Cm-246	5,27E+05	1,12E+06	2,12
Total	1,60E+14	4,61E+14	2,88

3.4 2BMA Vault

The proposed 2BMA inventory is presented in Table 3-4. A comparison with the reference inventory (SKB 2013) is also given in the same table. For individual waste packages, see Appendix B.

Table 3-4 2BMA Inventory

Radionuclide	Reference inventory [Bq]	Maximum inventory [Bq]	Uncertainty factor (1.645 σ)
H-3	3,31E+12	2,02E+13	6,09
Be-10	2,19E+04	9,85E+05	44,89
C-14 org	3,96E+09	6,89E+09	1,74
C-14 oorg	1,44E+10	2,40E+10	1,67
C-14 ind	5,09E+09	1,76E+10	3,46
Cl-36	2,02E+08	8,00E+08	3,96
Ca-41	1,56E+10	6,07E+10	3,90
Fe-55	1,05E+11	4,64E+11	4,44
Co-60	1,99E+12	3,74E+12	1,88
Ni-59	9,50E+11	2,12E+12	2,23
Ni-63	9,21E+13	2,02E+14	2,19
Se-79	7,29E+06	3,77E+08	51,78
Sr-90	3,60E+11	8,66E+11	2,41
Zr-93	1,06E+09	3,87E+09	3,66
Nb-93m	1,31E+13	2,50E+13	1,91
Nb-94	9,13E+10	1,80E+11	1,97
Mo-93	4,52E+09	8,65E+09	1,91
Tc-99	1,42E+09	5,00E+09	3,53
Pd-107	2,55E+09	4,72E+09	1,85
Ag-108m	4,07E+10	1,73E+11	4,25
Cd-113m	9,32E+07	4,15E+09	44,56
In-115	3,13E+05	8,29E+05	2,65
Sn-126	1,75E+07	7,46E+07	4,26
Sb-125	2,62E+08	1,27E+09	4,87
I-129	7,68E+06	4,50E+07	5,86
Cs-134	2,25E+08	3,30E+08	1,46
Cs-135	5,33E+07	3,22E+08	6,04
Cs-137	8,96E+11	4,58E+12	5,11
Ba-133	1,43E+08	5,30E+08	3,71
Pm-147	4,07E+08	8,60E+08	2,11
Sm-151	3,55E+10	1,36E+11	3,84
Eu-152	1,33E+11	4,78E+11	3,58
Eu-154	6,83E+09	1,93E+10	2,83
Eu-155	3,74E+08	9,72E+08	2,60
Ho-166m	5,22E+08	2,11E+09	4,04
U-232	1,46E+05	3,45E+05	2,36
U-234	3,04E+06	9,70E+06	3,19
U-235	7,82E+04	2,29E+05	2,93
U-236	6,00E+06	1,21E+07	2,02
U-238	1,23E+06	3,91E+06	3,18
Np-237	7,67E+06	1,58E+07	2,06
Pu-238	4,42E+10	9,11E+10	2,06
Pu-239	6,77E+09	1,49E+10	2,20
Pu-240	9,20E+09	1,97E+10	2,14
Pu-241	1,66E+11	3,51E+11	2,12
Pu-242	5,02E+07	1,03E+08	2,05
Am-241	4,13E+10	1,98E+11	4,80
Am-242m	1,83E+08	3,62E+08	1,97
Am-243	6,62E+08	1,31E+09	1,98
Cm-243	1,03E+08	2,12E+08	2,06
Cm-244	1,07E+10	2,15E+10	2,01
Cm-245	1,01E+07	2,00E+07	1,98
Cm-246	3,34E+06	6,50E+06	1,94
Total	1,13E+14	2,61E+14	2,30

3.5 1BTF Vault

The proposed 1BTF inventory is presented in Table 3-5. A comparison with the reference inventory (SKB 2013) is also given in the same table. For individual waste packages, see Appendix B.

Table 3-5 1BTF Inventory

Radionuclide	Reference inventory [Bq]	Maximum inventory [Bq]	Uncertainty factor (1.645 σ)
H-3	6,81E+07	3,41E+09	50,05
Be-10	1,37E+04	6,88E+05	50,09
C-14 org	9,84E+09	1,15E+10	1,17
C-14 oorg	1,89E+11	2,20E+11	1,17
Cl-36	1,44E+07	4,81E+07	3,34
Fe-55	8,34E+07	4,17E+08	5,00
Co-60	1,67E+10	1,71E+10	1,02
Ni-59	3,30E+10	1,03E+11	3,13
Ni-63	2,04E+12	6,43E+12	3,16
Se-79	1,57E+07	7,87E+08	50,15
Sr-90	3,48E+10	4,76E+10	1,37
Zr-93	2,29E+07	1,15E+09	50,09
Nb-93m	1,43E+09	2,87E+10	20,03
Nb-94	2,53E+08	1,27E+09	5,02
Mo-93	2,56E+08	3,50E+08	1,37
Tc-99	2,31E+09	8,31E+09	3,60
Pd-107	3,92E+06	1,57E+08	40,13
Ag-108m	1,51E+09	7,56E+10	50,08
Cd-113m	7,69E+07	3,86E+09	50,21
Sn-126	1,96E+06	7,87E+07	40,13
Sb-125	7,47E+06	7,47E+07	10,00
I-129	2,27E+07	9,05E+07	3,99
Cs-134	7,09E+04	8,54E+04	1,20
Cs-135	1,03E+08	2,38E+08	2,32
Cs-137	7,13E+11	8,07E+11	1,13
Ba-133	4,04E+06	8,11E+06	2,01
Pm-147	3,84E+06	7,68E+06	2,00
Sm-151	6,52E+09	1,33E+10	2,04
Eu-152	6,20E+07	1,24E+08	2,01
Eu-154	1,98E+09	4,03E+09	2,04
Eu-155	4,97E+07	1,00E+08	2,02
Ho-166m	8,79E+07	1,77E+08	2,02
U-232	1,62E+04	3,24E+04	2,00
U-234	9,86E+05	1,97E+06	2,00
U-235	1,84E+07	3,69E+07	2,00
U-236	4,02E+05	8,05E+05	2,00
U-238	8,55E+05	1,71E+06	2,00
Np-237	1,06E+06	2,13E+06	2,00
Pu-238	2,09E+09	3,95E+09	1,88
Pu-239	4,68E+08	8,38E+08	1,79
Pu-240	5,20E+08	9,01E+08	1,73
Pu-241	7,30E+09	1,46E+10	2,00
Pu-242	2,96E+06	5,92E+06	2,00
Am-241	6,14E+09	5,15E+10	8,38
Am-242m	7,34E+06	1,47E+07	2,00
Am-243	3,25E+07	5,54E+07	1,70
Cm-243	3,82E+06	7,55E+06	1,97
Cm-244	2,68E+08	5,19E+08	1,94
Cm-245	2,94E+05	5,90E+05	2,00
Cm-246	7,82E+04	1,57E+05	2,00
Total	3,06E+12	7,85E+12	2,56

3.6 2BTF Vault

The proposed 2BTF inventory is presented in Table 3-6. A comparison with the reference inventory (SKB 2013) is also given in the same table. For individual waste packages, see Appendix B.

Table 3-6 2BTF Inventory

Radionuclide	Reference inventory [Bq]	Maximum inventory [Bq]	Uncertainty factor (1.645 σ)
H-3	1,07E+08	5,37E+09	50,06
Be-10	2,48E+04	1,24E+06	50,11
C-14 org	6,07E+09	7,08E+09	1,17
C-14 oorg	2,69E+11	3,13E+11	1,17
Cl-36	1,66E+07	4,06E+07	2,45
Fe-55	1,14E+08	5,72E+08	5,00
Co-60	2,36E+10	2,41E+10	1,02
Ni-59	3,83E+10	1,16E+11	3,03
Ni-63	2,27E+12	6,88E+12	3,04
Se-79	1,54E+07	7,72E+08	50,00
Sr-90	5,76E+10	1,23E+11	2,14
Zr-93	4,14E+07	2,07E+09	50,11
Nb-93m	2,34E+09	4,70E+10	20,04
Nb-94	4,13E+08	2,07E+09	5,03
Mo-93	2,36E+08	3,22E+08	1,37
Tc-99	5,45E+08	6,44E+08	1,18
Pd-107	3,86E+06	1,54E+08	40,00
Ag-108m	2,21E+09	1,11E+11	50,11
Cd-113m	6,36E+07	3,18E+09	50,00
Sn-126	1,93E+06	7,71E+07	40,00
Sb-125	1,04E+07	1,04E+08	10,00
I-129	1,02E+07	1,19E+07	1,17
Cs-134	8,85E+04	1,06E+05	1,20
Cs-135	1,85E+07	2,16E+07	1,17
Cs-137	6,23E+11	6,33E+11	1,01
Ba-133	6,20E+06	1,25E+07	2,01
Pm-147	4,57E+06	9,14E+06	2,00
Sm-151	6,15E+09	1,23E+10	2,00
Eu-152	6,54E+06	1,31E+07	2,00
Eu-154	1,80E+09	3,60E+09	2,00
Eu-155	5,83E+07	1,17E+08	2,00
Ho-166m	1,59E+08	3,21E+08	2,02
U-232	6,74E+03	1,37E+04	2,03
U-234	4,55E+05	9,25E+05	2,03
U-235	1,11E+05	2,23E+05	2,00
U-236	3,55E+05	7,15E+05	2,01
U-238	8,75E+05	1,76E+06	2,01
Np-237	1,98E+06	4,05E+06	2,05
Pu-238	4,55E+08	5,95E+08	1,31
Pu-239	1,89E+08	2,59E+08	1,37
Pu-240	2,64E+08	3,62E+08	1,37
Pu-241	2,42E+09	4,89E+09	2,02
Pu-242	1,37E+06	2,77E+06	2,03
Am-241	1,83E+09	5,93E+09	3,24
Am-242m	3,21E+06	6,52E+06	2,03
Am-243	1,78E+07	2,35E+07	1,32
Cm-243	4,15E+05	7,86E+05	1,89
Cm-244	2,84E+07	3,42E+07	1,20
Cm-245	1,36E+05	2,76E+05	2,03
Cm-246	3,60E+04	7,32E+04	2,03
Total	3,30E+12	8,30E+12	2,51

3.7 1BLA Vault

The proposed 1BLA inventory is presented in Table 3-7. A comparison with the reference inventory (SKB 2013) is also given in the same table. For individual waste packages, see Appendix B.

Table 3-7 1BLA Inventory

Radionuclide	Reference inventory [Bq]	Maximum inventory [Bq]	Uncertainty factor (1.645 σ)
H-3	2,00E+08	1,01E+10	50,41
Be-10	6,52E+02	3,42E+04	52,37
C-14 org	7,92E+07	9,40E+07	1,19
C-14 oorg	4,02E+09	4,77E+09	1,19
Cl-36	2,17E+07	2,88E+07	1,33
Fe-55	8,78E+06	5,05E+07	5,75
Co-60	1,02E+09	2,00E+09	1,96
Ni-59	4,00E+09	1,27E+10	3,17
Ni-63	3,05E+11	9,59E+11	3,15
Se-79	4,00E+05	2,06E+07	51,55
Sr-90	7,42E+08	2,88E+09	3,88
Zr-93	1,09E+06	5,69E+07	52,37
Nb-93m	7,68E+07	1,66E+09	21,62
Nb-94	3,14E+07	1,95E+08	6,22
Mo-93	1,01E+08	1,39E+08	1,37
Tc-99	1,85E+09	6,52E+09	3,51
Pd-107	1,00E+05	4,14E+06	41,32
Ag-108m	1,94E+08	9,86E+09	50,92
Cd-113m	1,96E+06	1,04E+08	53,16
Sn-126	5,00E+04	2,07E+06	41,32
Sb-125	4,74E+05	5,55E+06	11,71
I-129	4,35E+05	1,34E+06	3,07
Cs-134	1,58E+04	4,06E+04	2,57
Cs-135	3,07E+06	6,00E+06	1,96
Cs-137	1,84E+10	3,15E+10	1,71
Ba-133	2,20E+05	5,90E+05	2,68
Pm-147	3,02E+05	9,07E+05	3,01
Sm-151	1,68E+08	3,97E+08	2,37
Eu-152	1,01E+08	2,21E+08	2,18
Eu-154	4,00E+07	1,03E+08	2,57
Eu-155	1,54E+06	4,45E+06	2,88
Ho-166m	4,18E+06	1,04E+07	2,49
U-232	2,34E+03	7,60E+03	3,25
U-234	1,33E+05	4,29E+05	3,23
U-235	2,98E+08	7,21E+08	2,42
U-236	3,99E+04	1,29E+05	3,23
U-238	7,33E+08	1,55E+09	2,11
Np-237	6,75E+04	2,08E+05	3,08
Pu-238	3,47E+08	1,12E+09	3,24
Pu-239	6,60E+07	2,12E+08	3,21
Pu-240	6,74E+07	2,15E+08	3,19
Pu-241	1,29E+09	4,22E+09	3,28
Pu-242	3,99E+05	1,29E+06	3,23
Am-241	5,23E+08	6,08E+09	11,62
Am-242m	1,02E+06	3,31E+06	3,24
Am-243	4,00E+06	1,27E+07	3,19
Cm-243	7,58E+05	2,46E+06	3,25
Cm-244	5,39E+07	1,75E+08	3,25
Cm-245	3,97E+04	1,28E+05	3,23
Cm-246	1,05E+04	3,40E+04	3,23
Total	3,39E+11	1,06E+12	3,12

3.8 XBLA Vaults

The proposed inventory for all XBLA vaults is presented in Table 3-8. A comparison with the reference inventory (SKB 2013) is also given in the same table. For individual waste packages, see Appendix B.

Table 3-8 XBLA Inventory

Radionuclide	Reference inventory [Bq]	Maximum inventory [Bq]	Uncertainty factor (1.645 σ)
H-3	1,94E+11	5,43E+11	2,79
Be-10	1,26E+03	2,49E+04	19,75
C-14 org	2,25E+08	1,21E+09	5,37
C-14 oorg	9,27E+08	4,98E+09	5,37
C-14 ind	1,19E+09	6,41E+09	5,37
Cl-36	4,60E+07	1,13E+08	2,46
Ca-41	3,91E+09	1,01E+10	2,57
Fe-55	4,45E+08	2,64E+09	5,93
Co-60	2,59E+10	1,66E+11	6,41
Ni-59	1,15E+10	6,62E+10	5,74
Ni-63	1,12E+12	6,57E+12	5,86
Se-79	5,94E+06	5,85E+07	9,85
Sr-90	2,40E+10	1,80E+11	7,51
Zr-93	2,95E+07	2,09E+08	7,11
Nb-93m	1,34E+11	8,69E+11	6,48
Nb-94	9,82E+08	6,15E+09	6,27
Mo-93	9,01E+07	3,09E+08	3,43
Tc-99	4,98E+08	3,06E+09	6,15
Pd-107	1,72E+06	1,39E+07	8,06
Ag-108m	1,53E+09	8,46E+09	5,54
Cd-113m	6,13E+06	1,79E+08	29,18
Sn-126	7,93E+06	5,58E+07	7,04
Sb-125	4,46E+06	3,03E+07	6,78
I-129	1,93E+06	1,45E+07	7,49
Cs-134	1,40E+06	9,25E+06	6,63
Cs-135	1,76E+08	1,21E+09	6,91
Cs-137	4,95E+11	3,45E+12	6,97
Ba-133	1,26E+07	3,33E+07	2,65
Pm-147	1,19E+06	5,51E+06	4,62
Sm-151	5,89E+09	1,58E+10	2,69
Eu-152	1,73E+10	4,57E+10	2,65
Eu-154	2,67E+08	8,01E+08	3,00
Eu-155	1,16E+07	3,48E+07	2,99
Ho-166m	9,03E+07	2,39E+08	2,65
U-232	9,35E+03	3,66E+04	3,92
U-234	4,38E+05	1,44E+06	3,29
U-235	3,23E+08	1,03E+09	3,17
U-236	2,06E+05	9,61E+05	4,65
U-238	1,77E+08	4,47E+08	2,52
Np-237	2,61E+05	1,06E+06	4,08
Pu-238	1,52E+09	6,22E+09	4,09
Pu-239	2,77E+08	1,10E+09	3,98
Pu-240	2,95E+08	1,23E+09	4,16
Pu-241	5,74E+09	2,36E+10	4,12
Pu-242	1,71E+06	7,01E+06	4,10
Am-241	1,94E+09	2,20E+10	11,37
Am-242m	4,84E+06	2,07E+07	4,28
Am-243	1,86E+07	8,02E+07	4,31
Cm-243	3,40E+06	1,40E+07	4,13
Cm-244	2,80E+08	1,26E+09	4,52
Cm-245	2,18E+05	1,01E+06	4,63
Cm-246	6,60E+04	3,27E+05	4,95
Total	2,05E+12	1,20E+13	5,86

3.9 All vaults

The proposed inventory for the whole repository is given in Table 3-9. This includes the following repository vaults: Silo, BRT, 1BMA, 2BMA, 1BTF, 2BTF, 1BLA and all XBLA vaults.

Table 3-9 Repository inventory

Radionuclide	Reference inventory [Bq]	Maximum inventory [Bq]	Uncertainty factor (1.645 σ)
H-3	3,52E+12	2,12E+13	6,04
Be-10	1,27E+06	6,38E+07	50,15
C-14 org	9,22E+11	1,08E+12	1,18
C-14 oorg	5,09E+12	5,97E+12	1,17
C-14 ind	1,65E+10	4,26E+10	2,59
Cl-36	1,54E+09	5,62E+09	3,66
Ca-41	1,95E+10	7,08E+10	3,63
Fe-55	2,91E+12	1,59E+13	5,48
Co-60	1,55E+13	2,87E+13	1,85
Ni-59	1,02E+13	2,99E+13	2,95
Ni-63	8,06E+14	2,36E+15	2,93
Se-79	1,31E+09	6,56E+10	50,19
Sr-90	4,66E+12	6,68E+12	1,43
Zr-93	6,19E+09	1,26E+11	20,43
Nb-93m	2,36E+13	6,81E+13	2,88
Nb-94	1,91E+11	5,83E+11	3,05
Mo-93	2,92E+10	5,95E+10	2,03
Tc-99	6,33E+10	2,29E+11	3,63
Pd-107	2,89E+09	1,79E+10	6,19
Ag-108m	2,97E+11	6,44E+12	21,67
Cd-113m	1,06E+10	5,55E+11	52,20
In-115	3,13E+05	8,29E+05	2,65
Sn-126	2,61E+08	7,24E+09	27,71
Sb-125	1,32E+11	1,54E+12	11,64
I-129	1,17E+09	3,63E+09	3,09
Cs-134	2,20E+11	5,86E+11	2,66
Cs-135	5,67E+09	1,23E+10	2,18
Cs-137	7,06E+13	1,08E+14	1,53
Ba-133	8,31E+08	2,05E+09	2,47
Pm-147	3,59E+11	1,18E+12	3,28
Sm-151	6,00E+11	1,38E+12	2,29
Eu-152	1,52E+11	5,26E+11	3,47
Eu-154	5,58E+11	1,55E+12	2,78
Eu-155	1,01E+11	3,20E+11	3,17
Ho-166m	9,09E+09	1,98E+10	2,17
U-232	8,96E+05	2,18E+06	2,43
U-234	4,75E+07	1,11E+08	2,34
U-235	6,57E+08	1,82E+09	2,77
U-236	2,59E+07	6,14E+07	2,37
U-238	9,52E+08	2,08E+09	2,19
Np-237	5,75E+08	1,68E+09	2,92
Pu-238	1,32E+11	2,95E+11	2,24
Pu-239	2,80E+10	5,60E+10	2,00
Pu-240	3,87E+10	7,70E+10	1,99
Pu-241	5,23E+11	1,24E+12	2,38
Pu-242	2,03E+08	4,74E+08	2,34
Am-241	2,33E+13	2,82E+14	12,10
Am-242m	5,79E+08	1,35E+09	2,33
Am-243	2,57E+09	4,93E+09	1,91
Cm-243	3,25E+08	6,86E+08	2,11
Cm-244	2,19E+10	5,22E+10	2,38
Cm-245	2,84E+07	6,83E+07	2,41
Cm-246	8,58E+06	2,08E+07	2,42
Total	9,70E+14	2,94E+15	3,04

References

Bäcklin A, 1990. Kontroll av SFR-avfall. Delprojekt mätteknik. Gammadata mätteknik AB. Available as SKBdoc 1400742.

Cronstrand P, 2005. Assessment of uncertainty to correlation factors, SKB R-05-76, Svensk Kärnbränslehantering AB, Stockholm.

Cronstrand P, 2007. Assessing uncertainty to correlation factors for ^{14}C , ^{36}Cl , ^{59}Ni , ^{63}Ni , ^{93}Mo , ^{99}Tc , ^{129}I , and ^{135}Cs in operational waste for SFR 1. Vattenfall Power Consultant AB, T-CKV 2007-030.

Forsyth R, 1997. The SKB Fuel Corrosion Programme. An evaluation of results from the experimental programme performed in the SNAB Hot Cell Laboratory, SKB TR 97-25, Swedish Nuclear Fuel and Waste Management Co, Stockholm.

Gordon A, 2014. Principer för styrning av kärnavfall, deponeringsstrategi inom DFR samt tillämpning av denna inför ansökan om utbyggnad av SFR. SKBdoc 1434623.

Ingemansson T, 2000a. Osäkerheter vid uppskattning av Sr-90 och aktinidinventarie i SFR 1. SKB R-00-22, Svensk Kärnbränslehantering AB.

Ingemansson T, 2000b. Aktinidfördelningen i SFR 1. SKB R-00-01, Svensk Kärnbränslehantering AB.

Lindgren M, Pettersson M, Wiborgh M, 2007. Correlation factors for C-14, Cl-36, Ni-59, Ni-63, Mo-93, Tc-99, I-129 and Cs-135 in operational waste for SFR1. SKB R-07-05, Svensk Kärnbränslehantering AB, Stockholm.

Lundgren K, 2010. Mo-93, Tc-99 och Cs-135: Uppskattning av aktivitet i driftavfall från svenska LWR, Clab och Studsvik. ALARA Engineering, 06-0031R.

SKB, 2013. Låg- och medelaktivt avfall i SFR - Referensinventarium för avfall 2013. R-13-37, Svensk Kärnbränslehantering AB.

Thierfeldt S, Deckert A, 1995. Radionuclides difficult to measure in waste packages. Brenk Systemplanung BS-Nr 9203-6, Aachen, Germany.

Viertel M, Lundgren K, 2012. Uppskattning av Mo-93, Tc-99, I-129 och Cs-135 i driftavfall – Uppdatering till och med 2011. Studsvik ALARA Engineering, 12-0020R.

Appendix A

A.1 Normally distributed random variables

A normally distributed random variable has the probability density function

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad \text{Eq 3}$$

where μ is the mean (and median) and σ^2 the variance of the distribution. The cumulative distribution function is

$$D(x) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x-\mu}{\sqrt{2}\sigma} \right) \right], \quad \text{Eq 4}$$

where erf is the error function. A value that with a probability of 95% is higher than or equal to a sample from the distribution is here called $x_{95\%}$, and is given by

$$x_{95\%} = \mu + \operatorname{erf}^{-1}(0.9)\sqrt{2}\sigma \approx \mu + 1.645\sigma. \quad \text{Eq 5}$$

The relative error at 95% confidence level is then

$$\frac{\Delta x}{x} = \frac{1.645\sigma}{\mu}. \quad \text{Eq 6}$$

A.2 Log-normally distributed variables

A log-normal distribution has the probability density function

$$P(x) = \frac{1}{\sqrt{2\pi}Sx} e^{-\frac{(\ln x - M)^2}{2S^2}}, \quad \text{Eq 7}$$

where e^M is the median, $e^{M+S^2/2}$ is the mean and $e^{2M+S^2}(e^{S^2} - 1)$ is the variance of the distribution. The cumulative distribution function is given by

$$D(x) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\ln x - M}{\sqrt{2}S} \right) \right]. \quad \text{Eq 8}$$

A value that with a probability of 95% is higher than or equal to a sample from the distribution is then given by

$$x_{95\%} = e^{M + \operatorname{erf}^{-1}(0.9)\sqrt{2}S} \approx e^M e^{1.645S}. \quad \text{Eq 9}$$

The relative error at 95% confidence level is then

$$\frac{\Delta x}{x} = e^{1.645S} - 1. \quad \text{Eq 10}$$

A.3 The sum of two continuous stochastic variables

A random variable Z from a continuous distribution that is the sum of two independent continuous stochastic variables X and Y , with probability density functions $P_X(x)$ and $P_Y(y)$, has a probability density function given by

$$P_Z(z) = (P_X * P_Y)(z) = \int_{-\infty}^{\infty} P_X(t) P_Y(z-t) dt. \quad \text{Eq 11}$$

A.3.1 Sum of two normally distributed variables

The sum of two normally distributed variables is also normally distributed with mean $\mu_Z = \mu_X + \mu_Y$ and variance $\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2$. The relative error at 95% confidence is

$$\left. \frac{\Delta z}{z} \right|_{95\%} = 1.645 \frac{\sqrt{\sigma_X^2 + \sigma_Y^2}}{\mu_X + \mu_Y}. \quad \text{Eq 12}$$

A.4 The product of two continuous stochastic variables

A continuous stochastic variable Z that is the product of two continuous stochastic variables X and Y , has a probability density function given by

$$P_Z(z) = \int_{-\infty}^{\infty} P_X(t) P_Y(z/t) \frac{1}{|t|} dt. \quad \text{Eq 13}$$

A.4.1 The product of two log-normally distributed variables

The product of two log-normally distributed variables is also log-normally distributed with

$$\begin{aligned} M_Z &= M_X + M_Y \\ S_Z^2 &= S_X^2 + S_Y^2 \end{aligned} \quad \text{Eq 14}$$

The relative error at 95% confidence is

$$\frac{\Delta z}{z} \Big|_{95\%} = e^{1.645 \sqrt{S_X^2 + S_Y^2}} - 1. \quad \text{Eq 15}$$

The product of one normally distributed and one log-normally distributed variable is a distribution with a probability distribution given by

$$P_Z(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{t}{S_X \sigma_{YZ}} e^{-\left[\frac{(\ln z/t - M_X)^2}{2S_X^2} + \frac{(t - \mu_Y)^2}{2\sigma_Y^2} \right]} \frac{1}{|t|} dt \quad \text{Eq 16}$$

This equation has no analytic solution.

A.5 Mapping a normal distribution onto a log-normal distribution

In order to find an analytic expression to estimate $z_{95\%}$ for a stochastic variable that is the product of a normally distributed and a log-normally distributed variable, we may map the normal distribution onto a log-normal distribution so that we then may use Eq 13 to find $z_{95\%}$. To get the same $x_{50\%}$ and $x_{95\%}$ the log-normal distribution must fulfill

$$M_X = \ln \mu_X$$

$$S_X = \frac{\ln x_{95\%} - \ln \mu_X}{x_{95\%} - \mu_X} \sigma_X, \quad \text{Eq 17}$$

so that the relative error at 95% confidence is

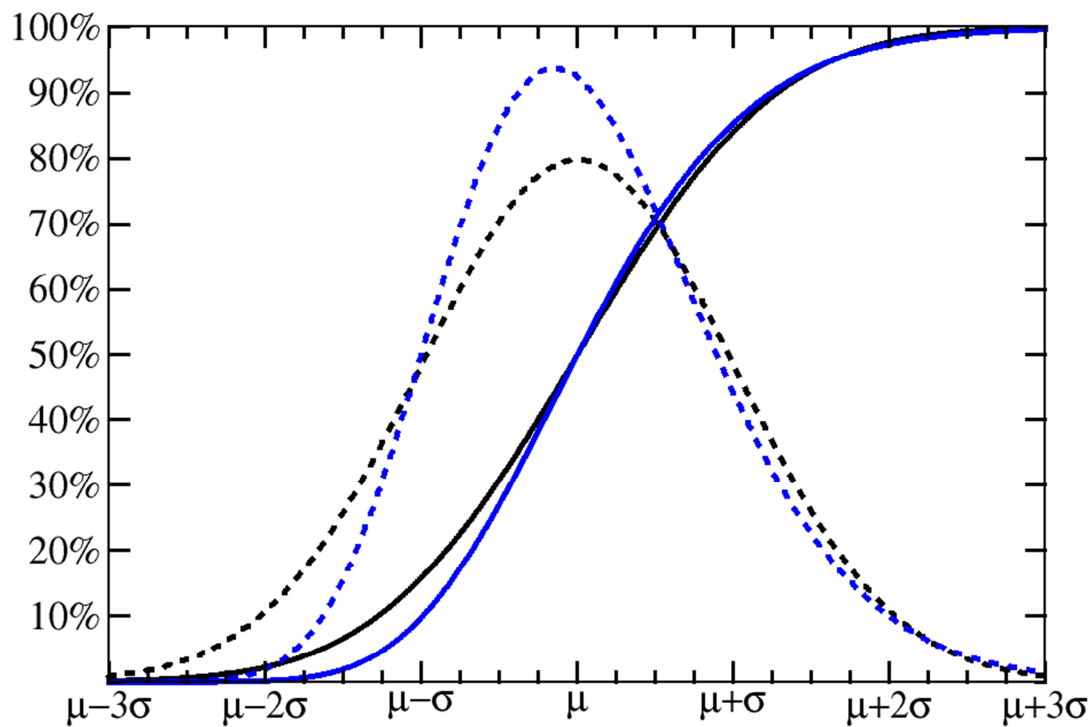
$$\frac{\Delta z}{z} \Big|_{95\%} = e^{\sqrt{\left[\ln \left(1 + 1.645 \frac{\sigma_X}{\mu_X} \right) \right]^2 + \left[\ln \left(1 + \frac{\Delta y}{y} \Big|_{95\%} \right) \right]^2}} - 1, \quad \text{Eq 18}$$

or

$$z + \Delta z \Big|_{95\%} = z \cdot e^{\sqrt{\left[\ln \left(1 + \frac{\Delta x}{x} \Big|_{95\%} \right) \right]^2 + \left[\ln \left(1 + \frac{\Delta y}{y} \Big|_{95\%} \right) \right]^2}}. \quad \text{Eq 19}$$

By matching the distributions at other confidence levels (at $k\sigma$) it can be shown that

$$z + \Delta z \Big|_k = z \cdot e^{\sqrt{\left[\ln \left(1 + k \frac{\Delta x}{x} \Big|_{1\sigma} \right) \right]^2 + \left[\ln \left(1 + k \frac{\Delta y}{y} \Big|_{1\sigma} \right) \right]^2}}. \quad \text{Eq 20}$$



Figur A-1 Mapping of a normal distribution onto a log-normal distribution, matching $x_{50\%}$ and $x_{95\%}$.